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Irish 14-Year-old Students' Knowledge of Initial Algebra

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Abstract: Initial algebra is a critical stage in the teaching of algebra and occurs when students are transitioning from arithmetic to algebra. Irish mathematics education at post-primary level has undergone a period of major reform beginning in 2010, which encompassed a radical change in the methods for teaching algebra. Despite this reform evidence emerged that students were struggling with algebra during their early years in post-primary school, however, no empirical based evidence of the specific content areas with which students struggled existed. This quantitative methods study aimed to establish a profile of what second year post-primary students (14 years old) in Ireland knew about algebra six years after the implementation of the reform. The evidence was collected using a standardised criterion referenced assessment known as a screener of initial algebra, which was developed and validated for use with Irish second year post-primary students. This research confirms that most students struggled with the key prerequisite content areas of fractions, decimal number magnitude, order of operations and exponents. Consequently, the algebra content items on variables, expressions and equations on the screener were not well answered by the majority which aligns with findings in previous international studies. The empirical results reported here are the first of their kind for Irish second-year post-primary students, providing valuable information for researchers and educators in Ireland as well as important evidence on students' knowledge of initial algebra in the international context, six years after curriculum reform and the introduction of new teaching approaches.

Keywords: Algebra, Initial algebra, Curriculum reform

Introduction

Five years after the reform of Irish mathematics post-primary curriculum in 2010, known as "Project Maths" numerous government reports, studies, and international testing revealed evidence of Irish students struggling with algebra (Chief Examiner, 2015; Shiel & Kelleher, 2017). Furthermore, Irish post-primary teachers had noted that several of the algebraic prerequisite content areas were difficult for students to master (Shiel & Kelleher, 2017). Since there was no empirical evidence in the Irish context, the purpose of this study was to create a profile of the knowledge that second-year post-primary Irish students had regarding algebra, specifically pre-requisite and initial algebra concepts.

Internationally, past research suggests a range of knowledge levels and misconceptions among post-primary school students in initial algebra (Booth et al., 2017). The U.S. National Mathematics Advisory Panel (NMAP) identified grade 8 (approx. 14 years old) as the latest time point to identify students who are struggling with algebra to allow sufficient time for intervention and remediation (Ketterlin-Geller & Yovanoff, 2009). Hence, this age group was targeted to address issues early.

Extensive international research has investigated various aspects of algebraic thinking among students from the age of six onwards demonstrating that algebraic concepts can be introduced early in primary school. This had prompted worldwide curriculum changes and the implementation of new teaching methods since the 1980s (Kieran et al., 2016). Mathematics education reform efforts over the past number of decades in many countries have led to a change in the method of teaching algebra. The method has shifted from the rule bound *transformational-based* approach to the *functions-based* approach which was introduced as part of a revised mathematics curriculum to all schools in Ireland in 2010 (Kieran, 2014; Prendergast & Treacy, 2017). A comparison of curricula in the USA showed that a key difference in the approaches was how the concept of a variable is introduced. In the transformational approach it is treated as a placeholder representing unknowns in expressions or equations, while in the functions-based approach variables are seen as varying quantities and representing relationships (Cai et al., 2010). This study is the first to examine students' knowledge of algebra in Ireland in light of the reform and introduction of the functions-based approach. There are few comparative studies investigating how algebra progresses in the curricula of different countries worldwide, therefore the results presented here will facilitate future international comparisons (Hemmi et al., 2020).

This study offers valuable insights into the specific difficulties Irish 14-year-old students encountered with algebra five years after the implementation of the functions-based approach, filling a gap as no detailed profile of their knowledge previously existed. Accordingly, it seeks to answer the question: *What do Irish 14-year-old students know about initial algebra?* By identifying prevalent misconceptions and errors, this research provides essential information for educators and researchers to better support student understanding in this foundational area.

Algebra and the Irish Post-primary Context

In Ireland post-primary education is completed in two cycles: junior and senior. The junior cycle lasts three years, typically starting at the age of 13. The average age of second year students is 14. The National Council for Curriculum and Assessment [NCCA] subject specification for junior cycle mathematics is defined in terms of overarching "statements of learning" with five contextual strands (NCCA, 2019). These strands are 1. Number, 2. Geometry and Trigonometry, 3. Algebra and Functions, 4. Statistics and Probability, together with a fifth unifying strand linking the learning in the other four strands (NCCA, 2017). Students build on the knowledge and proficiency they have developed from studying the 'Number' and 'Algebra" strands in primary school (NCCA, 2018). Subsequently, their engagement with patterns, relationships and expressions studied in first year of post-primary, should lay the foundations for algebra in second and third year.

For algebra the NCCA (2017) state that two aspects underlie all others: "algebra as a systematic way of expressing generality and abstraction, including algebra as generalised arithmetic, and algebra as syntactically guided transformation of symbols" (p. 26). These two aspects have led to the definition of three types of activities that students of algebra at junior cycle must engage in namely, representational activities, transformational activities and activities involving generalising and justifying.

Challenges with Initial Algebra

Algebra is widely recognised as a challenging area of mathematics due to its abstract nature and reliance on structural understanding and procedural fluency (Booth, et al., 2015). Persistent issues in algebra teaching and learning continue internationally, with ongoing research into measurement instruments to inform the area continue (Ketterlin-Geller & Yovanoff, 2009). Previous research shows that teachers often fail to recognise students' algebraic misconceptions which hinders students’ development with thinking algebraically and their progression with the subject as a whole (Asquith, Stephens, Knuth & Alibali, 2007).

This study contributes empirical data on Irish students’ understanding of initial algebra, offering insights into common misconceptions and supporting international comparisons, particularly in the context of a functions-based curriculum. Algebraic competence relies on a solid foundation in key prerequisite areas such as fractions, ratios, proportional reasoning, equality, variables, and functions (Bush & Karp, 2013). These topics align closely with Ireland’s junior cycle mathematics framework. Additionally, the new curriculum places added emphasis on pattern recognition and generalisation, which has been shown to support algebraic thinking (Warren & Cooper, 2008; NCCA, 2017).

Prior misconceptions in these foundational areas often persist despite instruction, and their influence fluctuates over the school year. While some errors, such as those involving fractions and mathematical properties, may decrease with instruction, misconceptions involving variables, equality, and negative numbers may increase (Booth et al., 2014). Relational understanding of equality and rational number knowledge have been identified as strong predictors of algebra success (DeWolf et al., 2015), while poor fraction understanding is linked to weaker algebra performance in Irish students (Shiel & Kelleher, 2017). Proportional reasoning is particularly significant for developing algebraic thinking, yet many students continue to rely on additive rather than multiplicative strategies (Hilton et al., 2013). This reasoning requires a conceptual understanding of ratios, equivalence, and relational structures, which adolescents find difficult to understand since it is a highly conceptual skill that takes time to develop (Singh, 2000). Other key challenges include the extension of number concepts to include negative numbers, which are commonly misunderstood both in representation and operation (Bush & Karp, 2013; Vlassis, 2008). Difficulties also emerge with indices, where students often apply incorrect procedural strategies due to surface-level understanding (Tall et al., 2001). Furthermore, misconceptions around the order of operations are widespread, with students frequently applying operations from left to right without recognising the underlying mathematical structure (Linchevski, 1995; Schwartzman, 1996). Linchevski (1995) advocates for allowing students to explore how different operation orders affect results, while Schwartzman (1996) argues that mnemonics are less effective than learning the hierarchy of operations naturally.

A conceptual understanding of variables, often confused with labels or placeholders, is essential for progress in algebra (Hunter, 2010; Stacey & MacGregor, 1997). Research shows that consistent exposure to algebraic notation and a clear explanation of variables as generalised quantities can mitigate such misconceptions (Bush & Karp, 2013). Errors in simplifying expressions, particularly failing to combine like terms, and detaching variables from operations are common (Kieran, 1992; Jupri et al., 2014). These foundational misunderstandings lead to difficulties

in solving equations, which requires both procedural fluency and conceptual understanding (Booth & Davenport, 2013). Students often memorise transformational rules without grasping their logic, especially regarding the preservation of equality during inverse operations (Capraro & Joffrion, 2006).
Chung and Delacruz (2014) identify three cognitive stages in algebraic equation solving: adaptability, adaptive expertise, and metacognition. Proficiency involves not only procedural skill but also a deeper understanding of structural relationships and flexibility in problem-solving approaches. Instructional strategies that integrate tables and graphs, as advocated by Carraher et al., (2006), may support these cognitive developments by reinforcing a functions-based understanding of equations.

In summary, the persistent challenges associated with initial algebra stem from deep-rooted misconceptions and gaps in foundational knowledge. Addressing these issues through conceptually focused instruction and continued research into student thinking is essential for improving algebraic understanding and long-term mathematical success.

Methods

This section outlines the research methodology employed in this quantitative study. The design and content of the screener is outlined, followed by the research samples, data collection and analysis.

Instruments

To profile second year post-primary students' (approx. age 14) knowledge of initial algebra a standardised criterion referenced screener was developed (Healy OBrien, 2021). The screener, a formative assessment, was designed to be of use to teachers in the Irish post-primary mathematics classroom. In the process of the development and validation of the screener over 500 students were assessed with a pen and paper version containing 21 task items that assessed the pertinent content areas as outlined in the appendix. Two types of item format were employed on this screener; multiple-choice known as selected-response (SR), and constructed-response, objective scoring (CROS) (Haladyna & Rodriguez, 2013). The SR items were a mix of conventional multiple choice where there was one correct answer, and complex multiple choice where there was more than one correct answer. The CROS items were open items where students responded with their workings and scored according to a rubric which detailed the criteria students must meet to receive a score of 0, 1 or 2.

In developing any measurement tool, the aim must be to reduce the measurement error, and to do this two important aspects were considered: validity and reliability (Hair et al., 2010). Assessment reliability refers to its ability to produce consistent results (Foster, 2017). Here stability, consistency over time, was the primary reliability measure used (Creswell, 2012). This was determined using the test-retest method, comparing scores from the October and April administrations. There was a strong positive correlation between the total score on the screeners at each administration at both individual and class level (OBrien & NíRíordáin, 2021).

The validity of the screener was established in terms of content, criterion, and construct validity (Foster, 2017). Research by Blanton et al., (2018) developed a conceptual framework for algebra; (1) generalised arithmetic; (2) equivalence, expressions, equations, and inequalities; and (3) functional thinking. As this model of algebraic thinking aligns closely with the junior cycle curriculum it was the framework employed to design and develop the screener. In using a conceptual framework aligned with the curriculum and the knowledge, skills and abilities (KSAs) outlined in the literature the *content validity* of the screener was established. "*Criterion validity* concerns how the measure compares with other measures" (Foster, 2017 p. 109). School based measures were compared with the results on the screener to establish criterion validity.

Construct validity is said to subsume all other types of validity and is the most disputed and difficult to establish "as it relates to what theories say about what a measure would be expected to look like" (Foster, 2017 p. 109). The construct of initial algebra was clearly defined using the conceptual framework outlined above. The screener was piloted in one school after which task items were revised to ensure the constructs were operationalised fairly. Classical test theory (CTT) looking at item difficulty and discrimination further established construct validity. The outline of the screener is presented in the appendix showing item number, type, score, and the KSAs assessed alongside the relevant component of the conceptual model and mapping to the Irish curriculum (O'Brien & Ní Ríordáin, 2017). The results of this study will be discussed in terms of these KSAs, providing a baseline profile of the students' knowledge in terms of their strengths and weaknesses.

Sampling

Irish second-year post-primary mathematics students were the study's population of interest which consisted of students enrolled in higher, ordinary, and foundation mathematics. For the academic year 2015–2016, there were 57,212 students enrolled in second-year post-primary education in Ireland, according to data from the Department of Education and Skills [DES], 2016. According to the sampling strategy employed by the Economic and Social study Institute (ESRI) study group, the post-primary school system is a natural clustering of second-year post-primary pupils (Murray et al., 2010). In September 2016 there were 735 post-primary schools listed in Ireland per the DES which provided the sampling frame (DES, 2016).

Post-primary education in Ireland is delivered in three types of schools namely, Secondary, Vocational, and Community and Comprehensive schools. Table 1 shows the breakdown of school types in the population and the sample. The co-educational status of a school formed part of the sampling frame adopted by Murray et al. (2010) and is therefore shown in Table 1. For this study no schools were excluded from all the post-primary schools in Ireland. All post-primary schools follow the same mathematics specification already outlined therefore one would not expect students' knowledge of initial algebra to vary based on their school type.

Table 1

Breakdown of population and sample by school type

School Type	**Number of Schools in the Population**		**Number of Schools in the Sample**		**Number of Students in the Sample**	
	n	**%**	**n**	**%**	**n**	**%**
Single-sex Girls Secondary	135	18.37	4	21.05	102	18.4
Single-sex Boys Secondary	103	14.01	3	15.79	130	23.4
Co-Ed Secondary	137	18.64	4	21.05	135	24.3
Subtotal	*375*	*51.02*	*11*	*57.89*	*367*	*66.1*
Single-sex Girls Vocational	2	0.27	0	0	0	0
Single-sex Boys Vocational	0	0.00	0	0	0	0
Co-Ed Vocational	263	35.78	7	36.84	165	29.7
Subtotal	*265*	*36.05*	*7*	*36.84*	*165*	*29.7*
Single-sex Girls Community/Comprehensive	1	0.14	0	0	0	0
Single-sex Boys Community/Comprehensive	1	0.14	0	0	0	0
Co-Ed Community/Comprehensive	93	12.65	1	5.26	23	4.1
Subtotal	*95*	*12.93*	*1*	*5.26*	*23*	*4.1*
Total	**735**	**100**	**19**	**100**	**555**	**100**

Due to its practicality and ability to facilitate multi-level analysis between students, classes, or both, a two-stage cluster sample design was used (Ross, 2005). The first step in this kind of design is choosing schools, and the second step is choosing a class, or cluster, of students within the school. Using a non-probability snowball sampling strategy, 29 teachers/classes and 19 schools were recruited (Creswell, 2012). As a result, 667 students made up the sample. The non-probability snowball sampling strategy, while necessary due to the practical difficulties of conducting research in educational settings, may have introduced some bias that could affect the generalisability of the findings to all Irish second-year post-primary students, however, the sample was almost representative of the population, including a range of school types and a distribution of students that is similar to the national distribution in terms of school type as shown in Table 1.

Students were assessed twice; in early October 2016 and in April 2017. The reason for this was two-fold, first to establish the reliability of the assessment using the test-retest method, and second to establish if there was any difference in the KSAs between the beginning and end of the academic year. Of the 667 students recruited data for 555 and 467 students was analysed from the October and April administrations respectively, after non-consent and absenteeism's were accounted for (Table 1). 305 (55.0%) of the 555 students who were assessed in October were male, 248 (44.7%) were female, and 2 (0.3%) chose not to disclose. In April, there were 212 (44.5%) females and 264 (55.5%) males. Compared to the 2016-2017 Irish post-primary gender distribution (47.7% male, 52.3% female), the sample overrepresented male students (Central Statistics Office, 2021). However, studies show that gender differences in mathematics ability have decreased with some suggesting that there are no differences which is further supported by the results here (Erturan & Jansen, 2015). Additionally, the sample overrepresents students from secondary schools (66.1% versus 51.02% in the population) and accordingly underrepresents those from vocational schools (29.7% versus 36.0%) and community/comprehensive schools (4.1% versus 12.9%). However, as the mathematics specification is uniform across all school types in Ireland and given that most community/comprehensive schools originated from the amalgamation of secondary and vocational schools, this underrepresentation is unlikely to have influenced the results.

Data Collection

Each school was visited, and key information and procedures were explained to participants. Consent/assent packs were distributed to all participants in compliance with ethics approval. Every student was given a unique identification number (UIN), which was documented by both me for their consent/assent, and the teacher for the purpose of administering the screener. The anonymity of the schools and students participating in the research was safeguarded by assigning a unique letter to each school and a UIN to each student. Screeners were posted on the same date in October 2016 and April 2017, with teachers administering them within a week and returning by post.

Data Analysis

Each student's screener responses, together with basic demographic, school, and class information, were included in the quantitative data collection. In order to create a profile of students' knowledge, quantitative data analysis was used to examine the results using CTT (Haladyna & Rodriguez, 2013). According to Anderson and Morgan (2008), a cognitive score (CS) is one that indicates a cognitive process, such as knowledge, recall, interpretation, or synthesis. Each response was subjected to a CS in order to analyse the data, yielding a screener score that may reach a maximum of 62. Items that were unanswered were noted as missing. The appendix contains an outline of the scoring system.

Additionally, each screener item's facility index (FI) was determined using the formula $FI = \frac{C}{N}$, where N is the total number of students in the sample and C is the number of students who properly answered an item. To determine which KSAs are comprehended by the majority of the sample, and which are not, the items were then sorted from highest to lowest by the percentage of respondents who answered entirely or partially correctly.

Results

This section discusses the findings which portray broad trends rather than deep explanation of Irish students' knowledge of initial algebra. Table 2 presents the items ranked in order by FI, from highest correct response rate to lowest in the October administration, creating a ranking, identifying which content areas are best and least understood. The content area of each item is listed alongside the response rates showing the percentage answered correctly or part correctly (%C), incorrectly (%I), or not answered (%N). The response rates for the April administration are listed alongside the October results and the increase/decrease in the percentage answered correctly or part correctly between administrations is also shown.

Table 2

Results from the administration of the screener in October 2016 and April 2017

Item	October 2016			April 2017			Change in %C
	%C	**%I**	**%N**	**%C**	**%I**	**%N**	
13 Equality – True/False answer	84.1	5.1	10.8	90.5	4.0	5.5	6.4
6.1 Proportional reasoning – True/False.	72.3	25.0	2.7	72.4	26.8	0.8	0.1
21.1 Patterns – complete first blank in a table.	71.8	14.1	14.1	78.6	16.4	5.0	6.8
16.1 Expressions – perimeter of a shape.	71.0	10.1	18.9	80.5	6.3	13.2	9.5
11 Comparing and ordering numbers.	64.7	25.4	9.9	74.8	19.3	5.9	10.1
10 Distributive property.	64.1	21.1	14.8	70.4	21.6	8.0	6.3
21.1 Patterns – complete blank in a table.	62.9	22.3	14.8	73.4	19.9	6.7	10.5
12 Equality – complete a number sentence.	56.8	22.0	21.2	71.7	15.3	13.0	14.9
2 Fractions and expressions	55.7	31.5	12.8	65.8	27.0	7.2	10.1
16.2 Expressions – perimeter of a shape.	50.3	27.6	22.1	59.5	26.4	14.1	9.2
6.2 Proportional reasoning – explanation	49.2	44.1	6.7	34.4	59.3	6.3	-14.8
21.2 Patterns – describe a pattern.	48.5	35.0	16.5	60.6	31.9	7.5	12.1
17 Expressions – simplify an expression.	46.8	33.3	19.9	56.0	30.6	13.4	9.2
19 Identify next step to solve a linear equation	45.6	37.1	17.3	56.8	35.0	8.2	11.2
4 Equivalent fractions	42.2	49.2	8.6	49.3	45.1	5.6	7.1
3 Fractions and expressions	38.4	45.0	16.6	45.7	46.5	7.8	7.3
1 Decimal number magnitude.	37.3	55.0	7.7	35.6	59.1	5.3	-1.7
7 Exponents and algebraic expressions.	35.9	47.4	16.7	32.7	58.1	9.2	-3.2
16.3 Expressions – perimeter of a shape.	35.9	39.5	24.7	50.9	33.1	15.9	15.0
18 Identify next step to solve a linear equation	34.4	46.7	18.9	44.2	45.3	10.5	9.8
15.1 Equations – solve a linear equation.	31.2	40.9	27.9	49.5	34.6	15.9	18.3
9 Order of operations.	29.5	59.6	10.8	36.1	57.9	6.1	6.6

21.3 Identify a formula for a pattern.	28.6	52.4	19.0	42.3	48.4	9.3	13.7
14 Variables and expressions.	22.7	68.6	8.7	25.2	69.2	5.6	2.5
20 Identify an equation for a given situation.	22.2	62.5	15.3	25.4	67.5	7.1	3.2
5 Patterns, fraction knowledge and expressions.	21.1	65.8	13.1	30.0	62.1	7.9	7.9
16.4 Expressions - perimeter of an open shape.	12.8	49.0	38.2	22.9	45.9	31.2	10.1
8 Exponents and algebraic expressions.	8.8	71.4	19.8	11.9	77.6	10.5	3.1

More than half of the sample provided correct or partially correct answers to questions about equality, proportional reasoning, patterns, and properties of numbers. These are discussed first. Items evaluating fractions, decimal number magnitude, order of operations, and exponents are then examined because fewer than half of the sample provided correct answers to these items. Students' achievement in the prerequisite content areas is then taken into consideration while discussing the algebraic content items of variables, expressions, and equation solving. Changes in the proportion of correct answers between the October and April administrations are classified as significant or not at the 5% level throughout the discussion. Where no p-value is reported the change in the proportions were not statistically significant. McNemar's test was run for the paired data (the students CS on an item in October versus April) on each dichotomously scored item to check if the difference in the proportion of the sample answering correctly was statistically significant at the 5% level. For polytomous items, the McNemar-Bowker test of symmetry was used for the paired data to examine the changes in the categorical responses between administrations. Where there was a significant difference, McNemar's test was run to identify which proportional change was significant. That is identifying if the change was due to the proportion answering partly correct now answering it fully correct or another combination. Results were then adjusted using Bonferroni correction to the α-level to control the overall Type 1 error rate when multiple significance tests are carried out (Field, 2009).

Equality, Comparing and Ordering Numbers, Proportional Reasoning, Properties of Numbers, and Patterns

Item 13, assessing equality, has the highest accuracy in both administrations. Item 12, also on equality, sees correct responses increase from 56.8% in October to 71.7% in April, a significant rise ($p = 0.001$). The concept of equality and the meaning of the '=' sign is firmly embedded in primary school in Ireland, where children from junior infants upwards are encouraged to explore equality using number balance (NCCA, 1999; NCCA, 2018). This study shows that most Irish students aged 14 have a solid grasp of equality. The rising proportion of students understanding the equals sign contrasts with Booth, Barbieri, et al. (2014), who found more errors with this symbol as the academic year progressed. Some students still struggle with equality, which is a strong indicator of overall performance. Those who answered item 12 incorrectly (9% in October, 5% in April) were among the lowest scorers, while none of the top scorers got it wrong.

Students' ability to explain their reasoning decreased from 49.2% correct or partially correct in October to 34.4% in April, despite the fact that 72.3% of them correctly answered the true/false portion of item 6 on proportional reasoning. This indicates a gap in their comprehension of a crucial component of junior cycle numeracy (NCCA, 2017). As stated, proportional reasoning is considered to be one of the main elements of formal thought and can help with learning in all areas of science, mathematics, and life and as such this result is worrying (Bush & Karp, 2013). Item 21, assessing patterns, showed 71.9% of students in October and 78.6% in April (not significant) correctly answering for the perimeter of 2 stacked hexagons (with image). When asked for the perimeter of 5 stacked hexagons without an image, correct answers rose from 62.9% in October to 72.3% in April, a significant increase ($p = 0.02$). When explaining how to find the perimeter of 100 hexagons, 48.5% answered correctly in October increasing to 60.6% in April, a significant improvement ($p < 0.001$). The ability to see patterns, explain them and represent them with algebraic symbolism is important for mathematical problem solving and given the functions-based approach these results are promising (Warren & Cooper, 2008).

Item 10 on the distributive property was answered correctly or partially correctly by a large proportion of students in both administrations. Item 11, on comparing and ordering numbers, was also well answered by most. While the distributive property is often reinforced through practice, this procedural knowledge doesn't necessarily indicate a full understanding of the concept. This is evident from the poor responses to item 18, which assessed the next step in solving an equation involving the distributive law. Less than half of the students correctly identified that multiplication by a negative sign distributes over the addition inside brackets (Mok, 2010). Furthermore, 71.4% of students in October, increasing to 77.6% in April, incorrectly expanded $(x-2)^2$ in item 8, indicating a lack of understanding of the distributive property.

Fractions, Decimal Number Magnitude, Order of Operations, and Exponents

All other items were answered correctly by less than half of the sample at both administrations. These items, which assess the content areas of fractions, decimal number magnitude, order of operations, exponents, variables, and equations give rise for concern. A small proportion of students correctly answered items 3, 4, and 5 indicating a concerning lack of understanding of fractions in both administrations, as shown in Table 2. These results align with Irish post-primary teachers' reports of students' deficient fraction knowledge (Shiel & Kelleher, 2017). Item 1 responses show a lack of knowledge of decimal number magnitude, which a US study found to be a strong predictor of algebra knowledge (DeWolf et al., 2015).

Fewer than 1 in 3 students answered item 9 on order of operations correctly in both administrations. The most common error was working left to right, made by 32.4% of students in October and 30% in April. Additionally, many students made errors with exponents, as shown by their responses to items 7 and 8 in Table 2. Item 8, the worst answered item, assesses the distributive law, expressions, and exponents. The two components of exponential notation—a base and an exponent—are used to express repeated multiplication. Exponents are acknowledged to be a challenging mathematical concept for students of all ages, and working with them requires a comprehension of its notation, meaning, and characteristics (Ulusoy, 2019).

Variables, Expressions, and Equations

Given the weak performance in key prerequisite areas, poor results on algebra items were expected. Item 14 on variables and most items on expressions were poorly answered, with only about half the sample correctly answering the simpler items (16.2 and 17). Only one item (16.1) on algebraic expressions was well answered in both administrations, with 71% in October and 80.5% in April answering correctly (not significant). This contrasts with 94% of UK students of a similar age in the 1978 ICAMMS study (Hodgen et al., 2009).

Item 16.4, which asked for an expression for the perimeter of an open-ended shape, saw correct answers rise from 12.8% in October to 22.9% in April which is significant ($p < 0.001$). This indicates improvement in handling algebraic expressions over the year. However, item 14 highlighted a persistent misunderstanding of variables as labels, with 39.3% in October and 45.9% in April making this error, which is comparable to the results from Küchemann's 1981 study. Despite decades of curricular reform and changes in teaching methods, this type of error has remained consistent internationally (Kieran et al., 2016; Prendergast & Treacy, 2017).

Equation solving is the essence of success with initial algebra. Procedural errors and misconceptions relating to variables and all other prerequisite content areas can hinder a student's ability to fluently solve equations (Bush & Karp, 2013; Kieran et al., 2016). For items assessing equations, items 15 and 19 showed a statistically significant improvement in correct answers from October to April, while items 18 and 20 showed no significant change. Item 20 asks students to select the correct equation to represent a given situation and only 1 in 5 in October, rising to 1 in 4 in April could answer this correctly.

Discussion

There was prior evidence indicating Irish post-primary students were having difficulty with algebra, but this evidence was retrospective in nature and did not focus on specific algebraic errors and misconceptions (Chief Examiner, 2015; Shiel & Kelleher, 2017). Despite the introduction of the functions-based approach, evidence here indicates that significant gaps in understanding important content areas persist for Irish students (Prendergast & Treacy, 2017). Overall, the results reveal variations in performance across different content areas. While students perform relatively well on items related to equality, proportional reasoning, patterns, properties of numbers, and simpler algebraic expressions, they struggle with items assessing fractions, decimal number magnitude, order of operations, exponents, variables, expressions, and equation solving. These findings align with previous studies but also reveal a nuanced understanding of the challenges when learning algebraic concepts and provide crucial information for educators and policymakers (Liang et al., 2018; Booth, Barbieri, et al., 2014; Bush & Karp, 2013). A key observation is the persistence of common misconceptions among students internationally despite different curricula and teaching methods. The insights provided by these results shed light on potential gaps in both instructional methods and curriculum design. A summary of the results is provided in Table 3 below for ease of reference.

Table 3

Summary of student performance across content areas

Content Area	**Item(s)**	**Key Observations**
Equality	12, 13	Strong grasp overall; item 13 is the best answered item. A lack of understanding of the equal sign is a strong indicator of overall poor performance.
Proportional Reasoning	6	Strong comparison skills as identified by the true/false part of the question indicating a solid understanding of equivalent fractions and ratio. However, students' ability to explain their reasoning decreased between administrations demonstrating a lack of conceptual understanding.
Patterns	21	Most are able to complete the table of missing values for the given pattern, with improvement between administrations. However, generalising from visual patterns still challenging for many, but there is significant improvement between administrations.
Properties of Numbers	8, 10, 18	Good procedural accuracy (Item 10), but poor application of distributive law evident in the responses to Items 8 and 18 indicating a poor conceptual understanding.
Comparing & Ordering Numbers	11	Well answered by most students.
Fractions	3, 4, 5	Less than half of the students answered these items correctly. Persistent weakness in foundational fraction knowledge which aligns with teacher reports.
Decimal Magnitude	1	Lack of decimal magnitude understanding; a known predictor of algebra success.
Order of Operations	9	Fewer than 1 in 3 students correctly answered this item. The most common error was working left-to-right with little improvement between administrations.
Exponents	7, 8	Poor understanding of exponent rules and notation; Item 8 was worst answered item overall.
Variables & Expressions	14, 16.1–16.4, 17	Persistent misconceptions viewing variables as labels evident in the responses to item 14. There is improvement in working with expressions between administrations.

Equations	15, 18, 19, 20	Some improvement in procedural fluency between administrations as evidenced by Items 15, 19. Item 20 assessing the reversal order error was poorly answered, further emphasising students' misconceptions with variables (viewing as labels).

Additionally, this research contributes key insights into 14-year-olds pre-pandemic knowledge of initial algebra. Post-pandemic Ireland ranked 11th worldwide for mathematical proficiency in the 2022 program for international student assessment (PISA), although the average score of 492 is a significant decline on the previous result of 500 in 2018 (İdil et al., 2024). Coşkun and Kara (2022) demonstrated that school closures and distance education during the pandemic negatively influenced mathematical skills. Furthermore, it has been reported that the shift to remote learning during the pandemic in Ireland posed considerable challenges for algebra instruction, with many teachers postponing its delivery until face-to-face teaching resumed (Cagney & Conway, 2021). Considering this, the evidence here is important both in Ireland and internationally in outlining what 14-year-olds knew about initial algebra prior to the pandemic and the measurement of possible learning loss due to school closures.
As discussed in the previous section and presented in Table 2, more than half of the sample exhibited knowledge of equality, proportional reasoning, comparing and ordering numbers, the distributive property, basic algebraic expressions, and patterns. Knowledge of these prerequisite content areas are necessary to develop a student's understanding of algebra (Bush & Karp, 2013). Measures introduced to help understand the relational knowledge of the equal sign, including the balance scale, have been evidenced in this study as effective for Irish students as the vast majority exhibit a strong understanding of this. To understand proportional reasoning fully a student must be able to: reason multiplicatively, understand rational numbers, analyse functional relationships, equivalence, ratio and its parts (Singh, 2000). The evidence here shows that the majority understand proportional relationships, although they are unable to explain them.

Students can encounter difficulties in comparing and ordering numbers when presented in different formats such as fractions, decimals, and percentages (Bush & Karp, 2013). Encouragingly, results here show that most can calculate a percentage and decimal of two whole numbers without a calculator and compare these values. Additionally, most can apply the distributive property, which is fundamental skill for algebra, given that it is used frequently in the transformation of expressions (Mok, 2010). However, while fluency in the transformational rules and symbol manipulation are evident from item 10, a fuller understanding of the structural properties is lacking as evidenced from item 8. Furthermore, the study of patterns introduced with the new curriculum has been effective based on the results here, with many able to identify a pattern and complete the table of values for item 21. Visualisation of geometric patterns and their generalisations is a recognised way to assist students understand variables in algebra (Wilkie & Clarke, 2016). However, the results here agree with a much earlier one by Stacey (1989) in that students are able answer the concrete questions about patterns but have difficulty in providing a general formula. While it is difficult to determine how students might have performed on such tasks prior to the curriculum change, the results suggest that the introduction of pattern-based questions has supported students' ability to work with concrete representations, even if generalisation remains a challenge. Students still struggle with establishing algebraic rules from patterns which is caused by the use of invalid methods to identify explicit formula, and this is confirmed in these results (Strømskag, 2015).

Areas of concern with prerequisite KSAs include fractions, decimals, exponents, and order of operations. Irish teachers had previously expressed concern about students' lack of proficiency with fractions on entering post-primary school, which negatively impacts algebra performance (Shiel & Kelleher, 2017). Findings here provide concrete evidence that Irish 14-year-olds do struggle with fractions, with common errors emerging relating to procedural knowledge of fractions and evidence of whole number misconceptions. Booth & Newton et al. (2014) found that understanding fraction magnitudes strongly predicts early algebra skills, including equation and word problem-solving. DeWolf et al. (2015) further concluded that decimal magnitude understanding, and relational knowledge of fractions are strong predictors of algebra performance. Responses to item 1 show that just less than 2 in 5 of our students understand decimal number magnitude.

Understanding exponents is required throughout algebra in both the transformational, generational and global/meta level skills (Bottoms, 2003; Bush & Karp, 2013). The poor responses to items 7 and 8 show a lack of understanding of exponents. Mastery of the correct order of operations is also essential, yet common misconceptions, like performing operations left to right, persist (Bottoms, 2003).

The poor performance on items assessing prerequisite skills predict struggles with variables, expressions, and equation solving (Booth & Newton et al., 2014; Bush & Karp, 2013). Mastery of these skills is crucial for success in initial algebra and prepares students for more advanced concepts (Capraro & Joffrion, 2006). Misconceptions in prerequisite areas can hinder problem-solving, obstruct learning new material, and persist despite targeted instruction (Booth et al., 2015). Research indicates that errors in different content areas fluctuate throughout the school year, specifically errors with fractions tend to decrease with instruction but errors involving variables, negative signs, and equality tend to increase (Booth & Barbieri et al., 2014). This study aligns with those findings, noting an increase in errors related to variables and exponents between administrations.

Conclusion

This paper profiles Irish14-year-old students' knowledge of initial algebra, highlighting key challenges and strengths at this critical stage of mathematical development. The findings emphasise the need to address common misconceptions in prerequisite skills to improve algebraic proficiency. However, this study has several limitations. It focuses solely on Irish students within a specific age cohort, assessed six years after the 2010 curriculum reform was introduced. Successful curriculum implementation typically requires years of sustained support, and at the time of this study, teachers were still adjusting to these changes (Johnson et al., 2019). Notably, the full implementation of the reform did not occur across all schools until June 2017, with many educators reporting a hybrid of traditional and reform-based approaches to teaching algebra during this period (Prendergast & Treacy, 2017). Subsequently, the revised Junior Cycle Mathematics specification was introduced in autumn 2018, building on the foundations of the Project Maths reform while shifting toward a more outcomes-based approach to algebra instruction (Byrne et al., 2021). As this study predates both this revised specification and the pandemic, it does not capture the effects of these significant developments.

Moreover, despite reforms promoting more student-led, conceptual approaches, recent findings indicate that direct instruction continues to dominate in many Leaving Certificate Higher Level classrooms, suggesting ongoing hesitancy or lack of confidence in adopting newer pedagogical models (Berry, Bray & Oldham, 2021). Further research is therefore needed to determine the extent to which the functions-based approach to algebra is being fully implemented in Junior Cycle classrooms. Cross-country comparisons would also be valuable to determine the effectiveness of the functions-based approach in diverse educational contexts, offering broader insights into algebra instruction globally.

Nonetheless, the findings of this study remain relevant even nine years after the data were collected and fifteen years since the initial reform. They provide a critical snapshot of the early outcomes of curriculum change and highlight foundational challenges and opportunities that continue to inform current practice. The persistence of certain trends observed in this study can serve as important reference points for evaluating longitudinal progress. Therefore, while this study offers valuable insights six years post-reform in the Irish context, future research should aim to replicate this study fifteen years post the initial curriculum reform and eight years after the junior cycle reform to assess long-term progress in algebraic understanding, particularly considering the impact of school closures due to the pandemic and the introduction of the revised Junior Cycle Mathematics specification.

As such, the screener developed as part of this research should be further developed for use as an online formative assessment tool by teachers in the Irish classroom. The data collected by teachers could then be easily collated and analysed to support future studies. This research shows that Irish students struggle with initial algebra as much as their international counterparts. The evidence presented shows common errors and misconceptions recognised in the literature and provides a much greater depth of understanding of Irish 14-year-olds difficulties than was previously known. It is hoped that this research will serve all stakeholders in mathematics education in understanding the strengths and weaknesses of students as they begin to grasp algebraic concepts. The information produced here can be used to determine how best to support teachers and students with initial algebra which is well documented as difficult to teach and learn (Demonty et al., 2018).

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Appendix

Item	CS			Item Type	KSA's assessed	Component of Conceptual Model	Junior Cycle Mathematics Subject Specification
1	0	1	2	Constructed Response	Decimal number magnitude.	Generalised arithmetic	*Number Systems*: - Use the number line to order natural numbers, integers, and rational numbers.
2	0	1	2	Complex Multiple Choice	Procedural fraction knowledge and algebraic expressions.	Generalised arithmetic	*Number Systems*: - Investigate models to think about operation on fractions. - Use the equivalence of fractions, decimals and percentages to compare proportions.
3	0		2	Conventional Multiple Choice	Procedural fraction knowledge and algebraic expressions.	Generalised arithmetic	*Number Systems*: - Investigate models to think about operation on fractions. - Use the equivalence of fractions, decimals and percentages to compare proportions.
4	0		2	Conventional Multiple Choice	Equivalent Fractions.	Generalised arithmetic	*Number Systems*: - Investigate models to think about operation on fractions. - Use the equivalence of fractions, decimals and percentages to compare proportions.
5	0		2	Conventional Multiple Choice	Relational fraction knowledge, algebraic expressions, and patterns.	Equivalence, expressions, equations, and inequalities	*Number Systems*: - Investigate models to think about operation on fractions. - Use the equivalence of fractions, decimals and percentages to compare proportions.
6.1	0		2	Conventional Multiple Choice	Proportional Relationships.	Generalised arithmetic	*Number Systems:* - consolidate their understanding of the relationship between ratio and proportion. Examining algebraic relationships: – proportional relationships
6.2	0	1	2	Constructed Response	Proportional Relationships.	Generalised arithmetic	*Number Systems:* - consolidate their understanding of the relationship between ratio and

							proportion. Examining algebraic relationships: – proportional relationships
7	0		2	Conventional Multiple Choice	Indices and algebraic expressions.	Generalised arithmetic	*Indices:* - use and apply the rules of indices
8	0		2	Conventional Multiple Choice	Indices and algebraic expressions.	Generalised arithmetic	*Indices:* - use and apply the rules of indices
9	0		2	Conventional Multiple Choice	Order of operations.	Generalised arithmetic	*Number Systems*: - Appreciate the order of operations, including use of brackets
10	0	1	2	Complex Multiple Choice	Properties of numbers.	Generalised arithmetic	*Number Systems*: - Investigate the properties of arithmetic and the relationships between them.
11	0		2	Conventional Multiple Choice	Comparing and Ordering Number, decimals, and percentages.	Generalised arithmetic	*Number Systems*: - - Use the equivalence of fractions, decimals and percentages to compare proportions.
12	0		2	Constructed Response	Equality.	Equivalence, expressions, equations, and inequalities	*Number Systems:* - Consolidate the idea that equality is a relationship in which two mathematical expressions hold the same value.
13	0		2	Conventional Multiple Choice	Equality.	Equivalence, expressions, equations, and inequalities	*Number Systems:* - Consolidate the idea that equality is a relationship in which two mathematical expressions hold the same value.
14	0		2	Conventional Multiple Choice	Variables and expressions.	Equivalence, expressions, equations, and inequalities	*Expressions*: - Using letters to represent quantities that are variable.
15.1	0		2	Constructed Response	Equations and integers.	Equivalence, expressions, equations, and inequalities	*Equations and inequalities*: - Selecting and using suitable strategies for finding solutions to equations and inequalities.
15.2	0		2	Constructed Response	Equations and integers.	Equivalence, expressions, equations, and inequalities	*Equations and inequalities*: - Selecting and using suitable strategies for finding solutions to equations and inequalities.
15.3	0		2	Constructed Response	Equations and integers.	Equivalence, expressions, equations, and inequalities	*Equations and inequalities*: - Selecting and using suitable strategies for finding solutions to equations and inequalities.
16.1	0		2	Constructed Response	Variables and expressions.	Equivalence, expressions, equations, and inequalities	*Expressions*: - Using letters to represent quantities that are variable.
16.2	0	1	2	Constructed Response	Variables and expressions.	Equivalence, expressions, equations, and inequalities	*Expressions*: - Using letters to represent quantities that are variable.

16.3	0	1	2	Constructed Response	Variables and expressions.	Equivalence, expressions, equations, and inequalities	*Expressions*: - Using letters to represent quantities that are variable.
16.4	0		2	Constructed Response	Variables and expressions.	Equivalence, expressions, equations, and inequalities	*Expressions*: - Using letters to represent quantities that are variable.
17	0		2	Conventional Multiple Choice	Expressions.	Equivalence, expressions, equations, and inequalities	*Expressions*: - Using letters to represent quantities that are variable.
18	0	1	2	Complex Multiple Choice	Equations.	Equivalence, expressions, equations, and inequalities	*Equations and inequalities*: - Selecting and using suitable strategies for finding solutions to equations and inequalities.
19	0	1	2	Conventional Multiple Choice	Equations.	Equivalence, expressions, equations, and inequalities	*Equations and inequalities*: - Selecting and using suitable strategies for finding solutions to equations and inequalities.
20	0		2	Conventional Multiple Choice	Equations.	Equivalence, expressions, equations, and inequalities	*Equations and inequalities*: - Selecting and using suitable strategies for finding solutions to equations and inequalities.
21.1	0		2	Constructed Response	Patterns.	Functional Thinking	*Representing situations with table diagrams and graphs:* - use tables, diagrams, and graphs as a tool for analysing relations
21.2	0	1	2	Complex Multiple Choice	Patterns.	Functional Thinking	*Representing situations with table diagrams and graphs:* - use tables, diagrams, and graphs as a tool for analysing relations
21.3	0	1	2	Complex Multiple Choice	Patterns.	Functional Thinking	*Representing situations with table diagrams and graphs:* - use tables, diagrams, and graphs as a tool for analysing relations
21	0		2	Constructed Response	Equations.	Equivalence, expressions, equations, and inequalities	*Equations and inequalities*: - Selecting and using suitable strategies for finding solutions to equations and inequalities.
Total	N/A		62				

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