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Teacher Interventions Using Guided Discovery and Mathematical Modelling in Grade 10 Financial Mathematics

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Abstract: Minimal guidance or maximal independence? How can teachers achieve that complex balance when teaching mathematics? In this pre-and post-test quasi-experimental research study, the authors explored the above broad questions in Grade 10 Financial Mathematics following guided discovery and mathematical modelling principles. Fifty-four (54) Grade 10 students at a Government High School in Johannesburg, South Africa, participated in the study. One group of students was taught following guided discovery and modelling principles and another group was taught the same contents but following explicit instruction. Students' mean scores were compared at the end of the interventions. Two teachers with similar experience and qualifications taught the two groups independently. The main findings revealed that although the overall performance in the post-test appeared similar in both groups, there was in fact a significant difference in understanding the concept of compound interest in the guided discovery and modelling group compared with the explicit instruction group. The research highlights the promise offered by the former teaching approaches over explicit instruction in supporting the understanding of difficult concepts to the students. Our findings led us to propose that foundation skills such as, plotting coordinate points, reading, and interpreting graphs, substituting into, and using formulas, and factorizing algebraic expressions should be thoroughly covered at the lower high school to prepare students to cope with more challenging related concepts at the higher level.

Keywords: *Guided discovery; mathematical modelling, financial mathematics; explicit instruction; quasi-experimental; Grade 10*.

Introduction

Background

The Curriculum and Assessment Policy Statement (CAPS) for the South African National Senior Certificate (NSC), Grade 10 to 12 Mathematics, outlines the curriculum and the aims for mathematics education at that level. The CAPS document takes mathematical modelling as "an important focal point of the curriculum" and emphasizes the need for real-life, contextual, and realistic problems that relate among others, to health, social, economic, cultural, scientific, and environmental issues (Department of Basic Education, 2011, p. 8). The inclusion of mathematical modelling in the curriculum is supported by research on the cognitive benefits of modelling. Blum (2015, cited by Wess, Klock, Siller & Greefrath, 2021). The researchers support teaching using modelling as it enhances students' understanding of real-world situations, reasoning, and critical evaluation of solutions. While modelling facilitates students' enjoyment of, and interest in mathematics, Blum (2015) cautions that the objectives of including modelling in mathematics education can only be met through high-quality teaching, which requires teachers to take a different approach from explicit teaching methods that they have been accustomed to over the years. Students may face challenges in the modelling cycle (Blum & Leiss, 2007) and require the intervention of a teacher to assist in the process. However, this guidance is not always clear to the teachers and to the learners as well (Blum & Ferri, 2009). For instance, how often should a teacher intervene and how should a teacher intervene to ensure that learning takes place with the learner taking more responsibility in the process?

This study explored the role of teachers in the mathematical modelling cycle with particular focus on teacher interventions and the results thereof. Teacher interventions were based on three guided discovery principles by De Jong & Lazonder (2014), undergirded by mathematical modelling principles (Blum 2015; Geiger et al .2022a; 2022b) to facilitate Grade 10 students' problem solving, and in turn improve their scores in financial mathematics and growth. We chose the topic of financial mathematics firstly because it is a modelling topic so directly promotes the objectives of the CAPS curriculum. Secondly, financial mathematics is an applied topic of mathematics because we know that students at grade 10 already have experience with money and can relate to the day-to-day real-life financial activities of buying and selling hence, they can understand the contextual meaning of financial growth in the curriculum. However, whereas the intentions of the CAPS curriculum are to promote learning through mathematical modelling, the topic of financial mathematics itself is often taught with little connection to real life examples. Pournara (2015) discusses the invisibility of the compound interest formula $F = P(1 + i)^n$ referring to a situation where the conceptual understanding and transparency of compound interest is overlooked, and the focus is only on the formula itself. In that case, students are only taught how to use the formula for substituting values in and getting out figures whose meanings they cannot explain. Understanding, and appreciation of the efficiency of the model, is only achieved when the derivation is understood. Modelling and guided discovery are such methods which can be used to help students with the derivation and deeper understanding of the formulae.

The purpose of this research study was thus to explore the three principles of guided discovery that teachers can use to assist students to construct knowledge whilst modelling mathematical problems in financial mathematics. The guided discovery principles by De Jong and Lazonder (2014) are heuristics, scaffolds, and direct presentation of information. Financial mathematics formulae are models of growth not understood by students, as students are not always taught the connection between procedure and concept. This research focused on the role of a teacher when students are guided to discover and model financial mathematics formulae by themselves. Fifty-four (54) Grade 10 students at a government high school in Johannesburg, South Africa, participated in the study. Two teachers at the high school volunteered to teach two grade 10 classes. One of the teachers at the school was presented with the three guided discovery principles and taught how to use these in the intervention class. The second teacher did not use the guided discovery principles, but direct instruction. The study adopted a pre-post-test approach to analyse the effects of teaching using the three guided discovery principles and modelling. The main finding was that although the overall mean scores in the post-test between the intervention and control groups were similar and therefore not significantly different, there was however one outstanding exception. We found a significant difference in the understanding of the concept of compound interest in the intervention group that followed the guided discovery learning approach (De Jong and Lazonder, 2014) than in the control group. The research therefore highlights the contributions of modelling and guided discovery learning in financial mathematics at grade 10. The finding gives much promise to adopting a modelling and guided discovery teaching approach when teaching challenging concepts in financial mathematics.

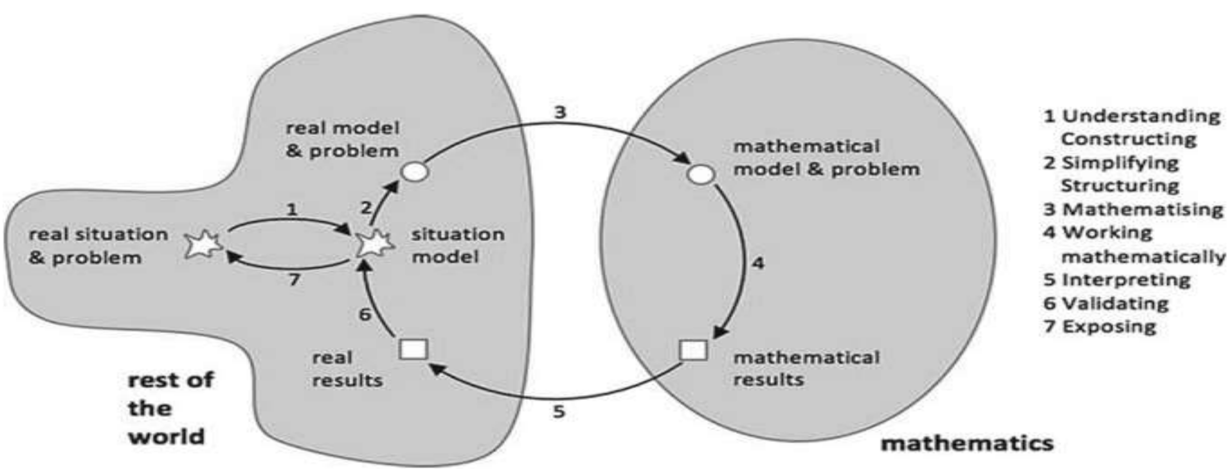
Teaching using modelling has been described as complex (Blum 2015; Wess et al. 2021, p. 11). Students may face challenges in the modelling cycle and require the intervention of a teacher to assist in the process. However, this guidance is not clear. How many times should a teacher intervene and how should a teacher intervene? The work of De Jong and Lazonder (2014) and Wess et al (2021) motivated us to test the guided discovery principles in a South African government school and to measure the efficacy of such principles. An additional objective of this research was to provide teachers with insight on how to intervene using guided discovery framework, which they can use to assist their learners in autonomous discovery. Teachers may be unaware of how to assist students to benefit from mathematical modelling. There may be reluctance from teachers to use modelling in their teaching because teaching using modelling is vastly different from many teachers' educational experiences (Asempapa & Sturgill, 2019). It would require supplementary knowledge to enable teachers to use a modelling approach. However, such new knowledge comes with the need for teachers to adjust, placing additional demands on an already demanding profession (Asempapa & Sturgill, 2019).

Mathematical Modelling

Mathematical modelling is a process that is aimed at understanding unstructured real-life situations. The goal of modelling a real-world situation is to enable the modeller to explain the situation and make some predictions based on the mathematical resolution of the situation (Geiger, Galbraith, Niss, & Delzoppo, 2022b). A modelling process involves identifying a problem within a real-world context, developing a relevant mathematical representation, determining a subsequent mathematical solution, interpreting the solution within the original context, and evaluating the solution's validity for resolving the problem (Blum, Galbraith, Henn, & Niss, 2007; Geiger et al., 2022a).

Figure 1

7 –Step Modelling Cycle adopted from Blum and Leiss (2007, p.225)



Being able to undertake all aspects of mathematical modelling in a holistic manner is interpreted as an indicator of one's modelling competency. Figure 1 by Geiger et al., (2022) is an ideal representation of the modelling sub-competencies (Blum & Leiss 2007) also endorsed in other research publications (e.g., Greefrath et al. 2013; Greefrath and Vorhölter 2016; Wess et al. 2021). Mathematical modelling and learning through modelling rely on the independence and autonomy of students, often complicating the traditional role of the teacher (Leiss & Wiegand, 2005). A research study conducted by Durandt et al.(2021) found that providing students with some autonomy and independence in problem solving can support their competency in mathematics. Moreover, there is also the benefit of developing a more positive attitude in mathematics in such groups than in the other groups that are taught without independence (Durandt et al., 2021). However, there are mixed results. For example, it has been reported (e.g., Mayer, 2004; De Jong & Lazonder, 2014) that as students are left to discover on their own, or with their peers, unguided discovery is generally ineffective. The same position is supported by Meyer's (1999) study based on group work where he highlighted the negative impact on student's autonomous work (Leiss & Wiegand, 2005). This group of studies suggests that to combat the potential negative impact of complete students' independence as well as minimise the complexity of modelling tasks, teachers should intervene and provide some guidance.

Distinguishing between Pedagogies

For purposes of this paper, we distinguish between implicit and explicit learning, and by extension implicit and explicit knowledge. By explicit learning, we refer to conscious operation where learning is structured (Ellis, Loewen, & Erlam, 2006) and directed by the teacher. Explicit knowledge is the knowledge that learners are consciously aware of and that is typically only available through controlled processing (Ellis et al. 2006). Implicit learning refers to a process of acquiring knowledge without much attention to structure and tends to take place without any conscious effort (Ellis et al. 2006). It is knowledge that students are only intuitively aware of and is accessible through automatic processing (Ellis et al. 2006). In this study, we assigned implicit teaching to the teacher who taught the intervention group (Ms.... Cameron), and explicit teaching was implemented (Ms.... Stanley). The three principles implemented in the intervention group under guided discovery all fall under implicit learning.

Guided Discovery

De Jong and Lazonder (2014) stress the importance of appropriate guidance, meaning guidance that will enable students to overcome cognitive obstacles whilst generating their own learning. The concept of learning by guided discovery is introduced by De Jong and Lazonder (2014), with the argument for its inclusion in science education being "material that is generated is better learned that material that is only received" (De Jong & Lazonder, 2014, p. 371). Other researchers (e.g. Hmelo-Silver, Duncan, and Chinn, 2007) also support the position that appropriate guidance can help overcome working memory constraints and promote the storage of new information in the learners. De Jong and Lazonder (2014) formulated six principles of guided discovery.

Process Constraints: Adding constraints to the initial problem and reducing possible outcomes to simplify the modelling and understanding process.

Performance Dashboard: A report that gives students an up-to-date view of their progression.

Prompts: Remind students when to undertake a particular task. Students have the capability of conducting such tasks however, they may not have been able to realise on their own that that was required.

Heuristics: A more specific guidance than prompts that provide students with actionable guidance, including assistance with procedure to enable a student to know how and when to perform a specific task.
Scaffolds: The structuring of a learning task into smaller, manageable units that link to each other to promote learning.
Direct Presentation of Information: Explicitly giving students the procedures and information when students have "insufficient prior knowledge" (De Jong & Lazonder, 2014, p. 378). A study by Alfieri, Brooks, Aldrich and Tenenbaum (2011) found that students who were taught using direct instruction produced better results than those who learnt through unguided discovery. This finding suggests that if modelling and sufficient guidance are used as tools to teach mathematics the students' scores can positively improve. In a related study by Furtak, Seidel, Iverson, and Briggs (2012), the researchers observed that the mean scores of students who received guidance in their discovery were larger than those who learnt through unguided discovery and direct instruction. These findings informed our research questions.

Research Questions

The main question that guided this study is: What is the impact of modelling and guided discovery teaching approach on grade 10 students' performance in financial mathematics? The sub-question is: Is there a difference in performance between grade 10 students who are taught based on modelling and guided discovery and students who are given explicit instructions by the teachers? To answer these questions, we compared students' performance in the post-test assessments.

Methods

Research Design

The study takes a quantitative approach where students' response to interventions are monitored by their scores in formative and summative assessments. The paradigm selected for this research is pragmatism. This is due to the practical nature of the research, providing concrete suggestions toward enhancing teaching using mathematical modelling and guided discovery. A quasi-experimental pre-post-test design was used to quantify the effects of teacher interventions on the test group.

Research Procedures

Two high school teachers volunteered to teach the two grade 10 classes, one in the intervention group, and the other in the control group. The teacher in the intervention class (hereafter Ms. Cameron) was presented with the three guided discovery principles and shown how to use them in her class. The second teacher, (hereafter Ms... Stanley), did not use the guided discovery principles, but applied the direct instruction method that she had been using before. The first researcher met with the teacher in intervention class, Ms. Cameron, to explain the purpose of the research and provide information on her role. Ms. Cameron was provided with the context of the research, why the students were using modelling tasks as the learning exercise, and guided discovery principles. Ms. Cameron was also given information on how to implement such interventions in the context of the given modelling task. It would have been unrealistic to introduce all six guided discovery principles to Ms. Cameron for the first time and expect her to apply all six effectively within a short period. For those reasons, we narrowed down to only the three guided discovery principles described above.

Participants

Fifty-four (54) Grade 10 students aged between 15 and 16 years at a government high school in Johannesburg, South Africa, were purposively selected and participated in the study. The students were already divided by their school into two teaching classes of 28 and 26 and we adopted the groups as they were without disrupting the school arrangement. Two high school teachers mentioned above taught the two grade 10 classes. The study was conducted with participants who are under the age of eighteen and so fall under one of the vulnerable categories. Therefore, to ensure that our research was ethically sound, parents received and signed consent forms on behalf of their students and students signed assent forms for them to participate in the research project. Participation was voluntary, and no learner was disadvantaged in any way by not taking part in the study. No incentives, for instance marks, were awarded for participation.

Design of Test Instruments

Pre-Test Design

The design of the pre-test items was aimed at assessing students' prior knowledge on financial mathematics and percentage increase whilst also assessing whether students had the competency to cope with the modelling tasks. These skills included interpreting a straight-line graph, determining the equation of a linear function, as well as factorisation by taking out the highest common factors. The four categories of questions were presented as shown below.

Table 1

The Four Categories of Skills Tested in the Pre-test.

Questions	Category
Questions 1, 2 and 3	1) Percentage increase and financial mathematics.
Questions 4 and 8	2) Interpreting a straight-line graph,
Questions 5, 6 and 7	3) Determining the equation of a linear function.
Questions 9 and 10	4) Factorisation by taking out the highest common factor.

Note: Table showing the four categories of skills assessed in pre-test for both groups.

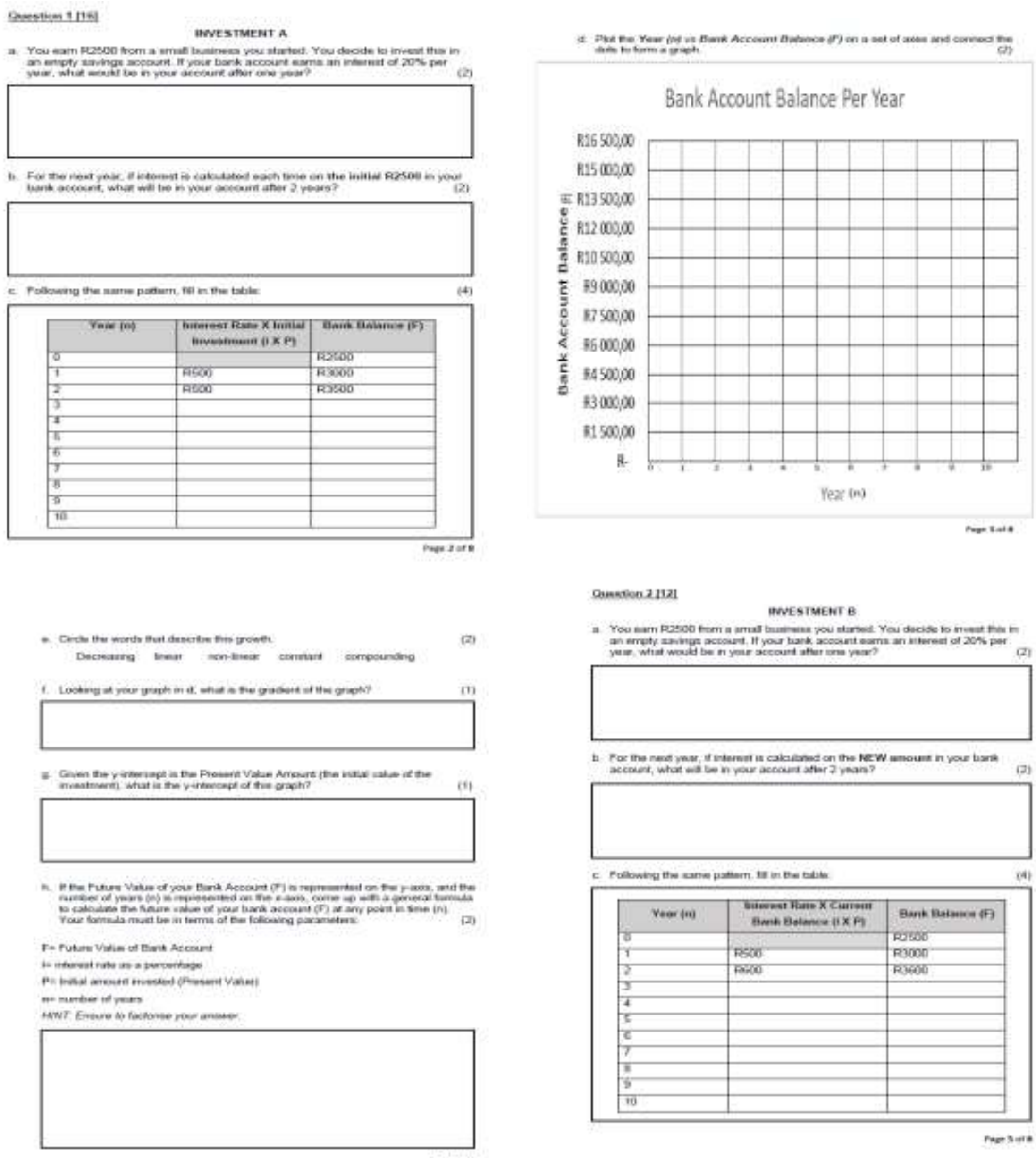
The test was assessed out of 20 marks. 10 questions were closed ended and went for, on average, two marks each. The coding of all three test instruments was split into the following categories: 1) Mechanical/procedural fluency, 2) Conceptual understanding, and 3) Application of learnt skills. The pre-test included mostly computation skills questions in order to assess learners' competencies.

Modelling Task Design

The modelling task was an investigation of simple interest and compound interest, the difference between the two types of interest and the relationship between simple interest and linear functions. Learners were tasked to investigate the effect of changing the interest calculation. The two aims of the task were for learners to derive the simple interest formula using linear functions and factorisation, as well as note the difference between compound and simple growth. The skills required for the second portion was plotting coordinate points and percentage increase.

Figure 2

Snippets of the Modelling Tasks



Question 1 [16]

INVESTMENT A

a. You earn R2500 from a small business you started. You decide to invest this in an empty savings account. If your bank account earns an interest of 20% per year, what would be in your account after one year? (2)

b. For the next year, if interest is calculated each time on the **initial R2500** in your bank account, what will be in your account after 2 years? (2)

c. Following the same pattern, fill in the table: (4)

Year (n)	Interest Rate X Initial Investment (I X P)	Bank Balance (F)
0		R2500
1	R500	R3000
2	R500	R3500
3		
4		
5		
6		
7		
8		
9		
10		

Page 2 of 8

d. Plot the *Year (n)* vs *Bank Account Balance (F)* on a set of axes and connect the dots to form a graph. (2)

Page 3 of 8

e. Circle the words that describe this growth. (2)

Decreasing linear non-linear constant compounding

f. Looking at your graph in d, what is the gradient of the graph? (1)

g. Given the y-intercept is the Present Value Amount (the initial value of the investment), what is the y-intercept of this graph? (1)

h. If the Future Value of your Bank Account (F) is represented on the y-axis, and the number of years (n) is represented on the x-axis, come up with a general formula to calculate the future value of your bank account (F) at any point in time (n). Your formula must be in terms of the following parameters: (2)

F= Future Value of Bank Account

I= interest rate as a percentage

P= Initial amount invested (Present Value)

n= number of years

HINT: Ensure to factorise your answer.

Page 4 of 8

Question 2 [12]

INVESTMENT B

a. You earn R2500 from a small business you started. You decide to invest this in an empty savings account. If your bank account earns an interest of 20% per year, what would be in your account after one year? (2)

b. For the next year, if interest is calculated on the **NEW amount** in your bank account, what will be in your account after 2 years? (2)

c. Following the same pattern, fill in the table: (4)

Year (n)	Interest Rate X Current Bank Balance (I X P)	Bank Balance (F)
0		R2500
1	R500	R3000
2	R600	R3600
3		
4		
5		
6		
7		
8		
9		
10		

Page 5 of 8

Note: The contents of Figure 2 are elaborated in the Results section.

Post-Test Design

The post-test items were designed to test the understanding of concepts discovered in the modelling tasks. The purpose was to quantify the effectiveness of the guided discovery principles used in the intervention group. The test items. included students' understanding of simple interest (linear growth), compound interest (exponential growth) and the fundamental difference between the two types of financial growth. A similar format to the pre-test categorisation was followed as below in Table 2.

Table 2

The Categories of Questions Assessed in the Post-Test

Questions	**Category**
Questions 1, 2, 3 and 4	1) Understanding simple interest
Questions 5, 6, 7 and 8	2) Understanding compound interest
Questions 9 and 10	3) Comparing simple and compound interest

Note: Table showing *the* categories of questions in the post-test. Categorisation is done by the authors.

The post-test was comprised of 10 questions and totalled 27 marks. The two last questions were open-ended. These questions were:

a. *If you were given the option to invest the same amount, at the same interest rate, but you could choose either simple interest or compound interest, which would you choose and why?*
b. *What is the main difference between simple interest and compound interest?*

These two questions fall under the code "conceptual understanding." Questions 1, 2, 5 and 6 were coded as "application of learning" whilst questions 3, 4, 7 and 8 were coded as "mechanical/procedural fluency" as students were required to substitute into a formula as well as algebraically manipulate a formula to solve for an unknown.

Data Collection and Analysis

Quantitative methods involving descriptive and inferential statistics were applied to analyse assessment scores and sample questionnaires. Data collection took place at five different points during the study: i. the students wrote the pre-test with pen and paper at the beginning of the intervention period and the pre-test scores were recorded for analysis; ii. Formative assessments were also given to students during the intervention period; iii. At the end of the interventions, both groups of students wrote the post-test, and the mean score for each question was used in the analysis; iv. The teacher in the intervention class (Ms. Cameron) completed a printed copy of a questionnaire about her experience implementing the three-guided discovery principles in her classroom; v. thirteen students, in each group were randomly selected to fill in a student questionnaire on their learning experiences. Due to space limitations, we focus our report in this paper on the analysis of pre-and post-test quantitative data, and only briefly refer to the other sets of data to provide additional evidence to our findings.

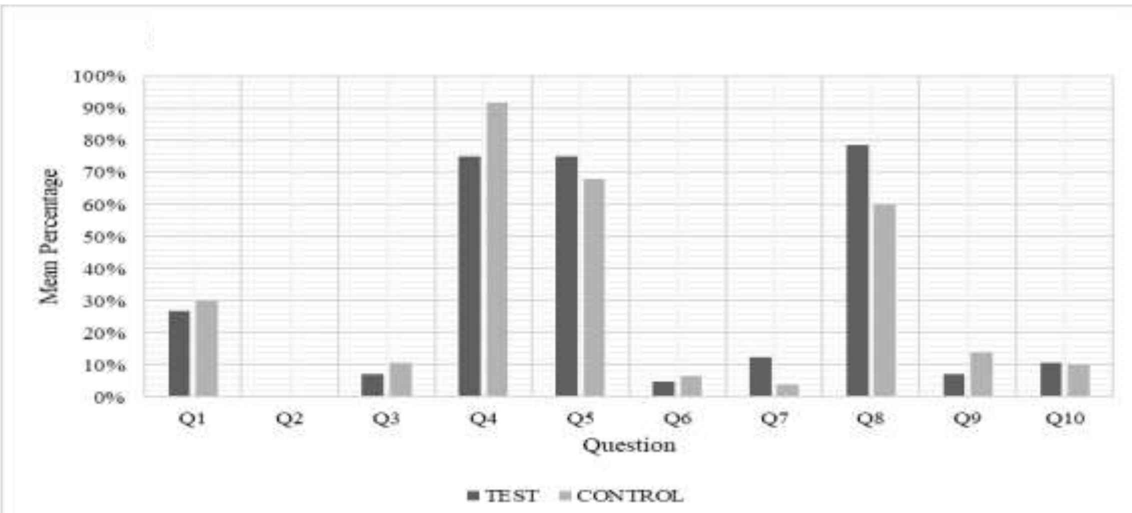
Results

Pre-Test Results

As shown in the graph in Figure 2 below, both groups performed similarly in the pre-test. Notable low performance in both groups for questions involving percentage increase (Q3), obtaining equations of linear functions (Q6 and Q7), and factorisation of functions by taking out the highest common factor (Q9, and Q10) can be observed from the graph in Fig.2. From these observations, at the beginning, questions in all four categories proved challenging to the students in both groups. It is not surprising that that the pre-test scores in both groups were in general low and not remarkably different from the other.

Figure 3

Graph Showing Performance in Pre-Test Items. by the Intervention and Control Group



Note: Graph comparing the performance of the intervention group and the control group in each question in the pre-test items. In Fig. 2, the Intervention group is labelled TEST, and the control group is labelled CONTROL.

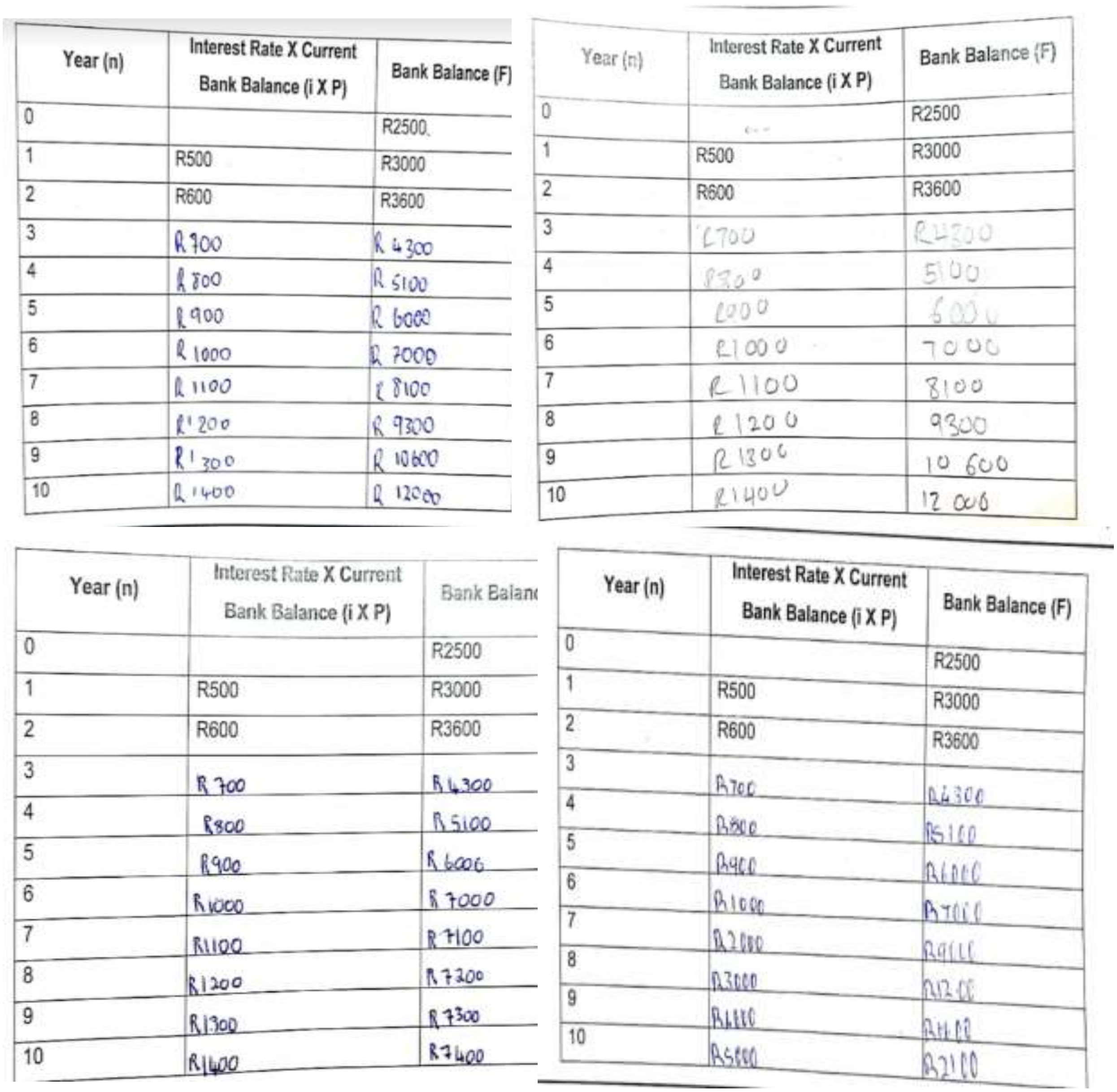
Modelling task data

It was clear, when reviewing the modelling tasks in each group, that students had similar misconceptions, however these misconceptions were not rectified in the control group. Below is a sample of students' responses to modelling tasks in the pre- and post-test respectively.

Students, in both test and control group, misunderstood the pattern of the compound interest model and were thus unable to solve the question correctly. Students could not process the compound increase in a bank account after each period and because of this lack of connection, could not justify to themselves that an increment of R100 (approximately $5) in the interest earned each period did not make sense. This change in direction would have been guided by the teacher. Ms.... Stanley had not guided her students in the change and, although the students did discover a pattern, it was not the correct one and created a gap in their understanding of compound interest. This gap is evident in the results of the post-test. This finding links Zulkarnaen's (2018) work about students not understanding a problem, particularly a modelling problem. He states that students "are likely to make a guess without having any mathematical thinking process" (p.5). However, after the interventions, some students in the test group showed improvement in the respect of the compound interest modelling task. Figures 4 and 5 are samples of students' responses to the compound interest question.

Figure 4

Samples of Answers to Compound Interest Question from Students in the Control Group.



Year (n)	Interest Rate X Current Bank Balance (i X P)	Bank Balance (F)
0		R2500
1	R500	R3000
2	R600	R3600
3	R700	R4300
4	R800	R5100
5	R900	R6000
6	R1000	R7000
7	R1100	R8100
8	R1200	R9300
9	R1300	R10600
10	R1400	R12000

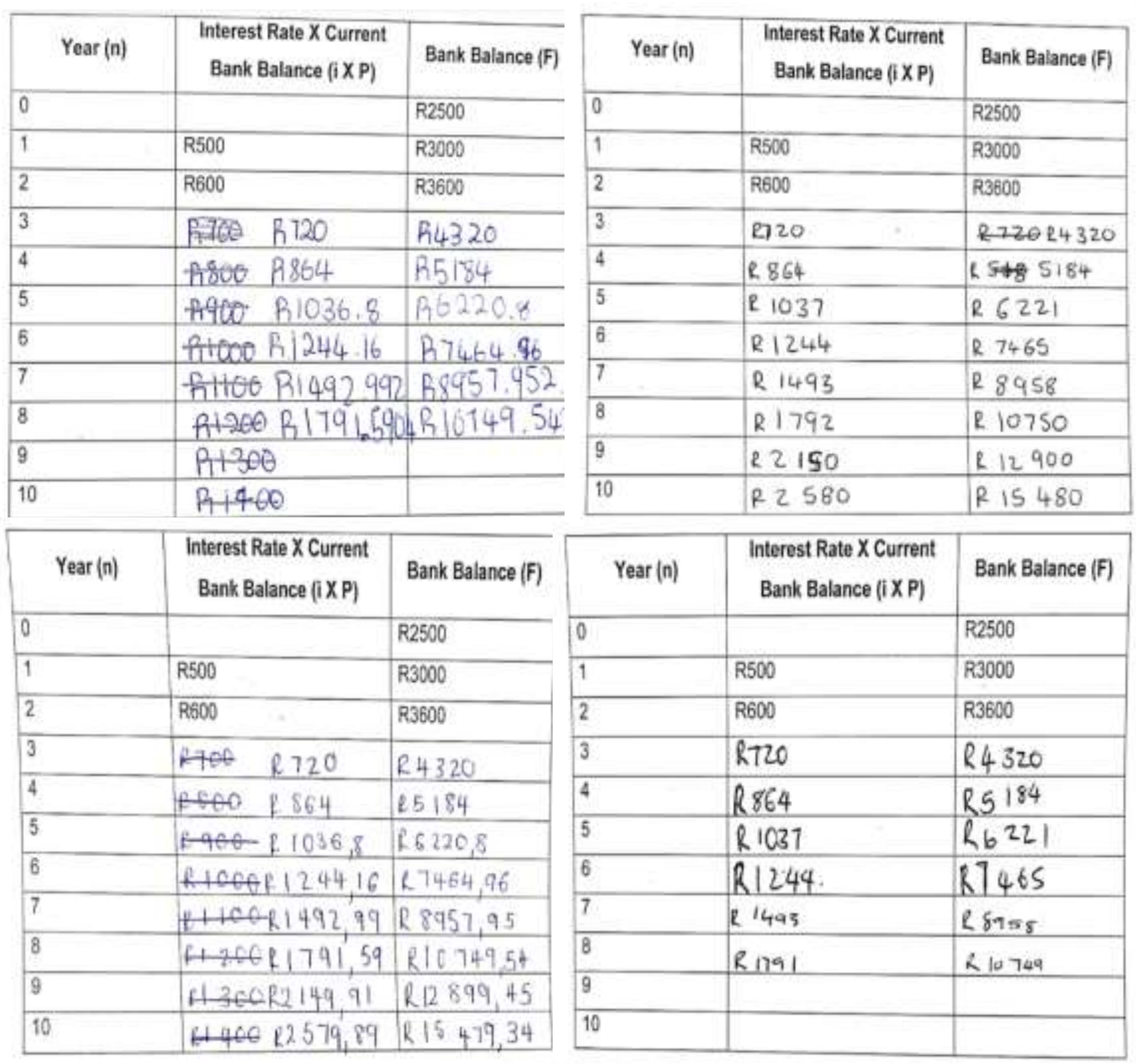
Year (n)	Interest Rate X Current Bank Balance (i X P)	Bank Balance (F)
0		R2500
1	R500	R3000
2	R600	R3600
3	R700	R4300
4	R800	5100
5	R000	6000
6	R1000	7000
7	R1100	8100
8	R1200	9300
9	R1300	10 600
10	R1400	12 000

Year (n)	Interest Rate X Current Bank Balance (i X P)	Bank Balanc
0		R2500
1	R500	R3000
2	R600	R3600
3	R700	R4300
4	R800	R5100
5	R900	R6000
6	R1000	R7000
7	R1100	R7100
8	R1200	R7200
9	R1300	R7300
10	R1400	R7400

Year (n)	Interest Rate X Current Bank Balance (i X P)	Bank Balance (F)
0		R2500
1	R500	R3000
2	R600	R3600
3	R700	R4300
4	R800	R5100
5	R900	R6000
6	R1000	R7000
7	R2000	R9000
8	R3000	R1200
9	R4000	R1400
10	R5000	R2100

Figure 5

Samples of Answers to Compound Interest Question from Students in the Test Group.



Year (n)	Interest Rate X Current Bank Balance (i X P)	Bank Balance (F)
0		R2500
1	R500	R3000
2	R600	R3600
3	~~R700~~ R720	R4320
4	~~R800~~ R864	R5184
5	~~R900~~ R1036.8	R6220.8
6	~~R1000~~ R1244.16	R7464.96
7	~~R1100~~ R1492.992	R8957.952
8	~~R1200~~ R1791.5904	R10149.54
9	~~R1300~~	
10	~~R1400~~	

Year (n)	Interest Rate X Current Bank Balance (i X P)	Bank Balance (F)
0		R2500
1	R500	R3000
2	R600	R3600
3	R720	~~R720~~ R4320
4	R 864	R ~~548~~ 5184
5	R 1037	R 6221
6	R 1244	R 7465
7	R 1493	R 8958
8	R 1792	R 10750
9	R 2150	R 12 900
10	R 2580	R 15 480

Year (n)	Interest Rate X Current Bank Balance (i X P)	Bank Balance (F)
0		R2500
1	R500	R3000
2	R600	R3600
3	~~R700~~ R720	R4320
4	~~R800~~ R864	R5184
5	~~R900~~ R1036,8	R6220,8
6	~~R1000~~ R1244,16	R7464,96
7	~~R1100~~ R1492,99	R8957,95
8	~~R1200~~ R1791,59	R10749,54
9	~~R1300~~ R2149,91	R12 899,45
10	~~R1400~~ R2579,89	R15 479,34

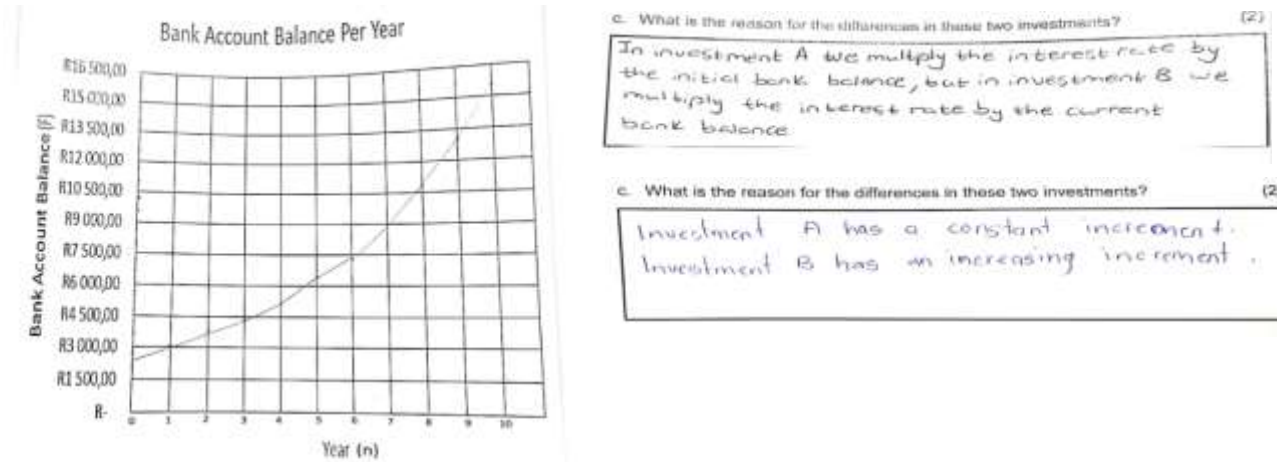
Year (n)	Interest Rate X Current Bank Balance (i X P)	Bank Balance (F)
0		R2500
1	R500	R3000
2	R600	R3600
3	R720	R4320
4	R864	R5184
5	R1037	R6221
6	R1244.	R7465
7	R1493	R8958
8	R1791	R10749
9		
10		

Post-Test Data

Apart from Q4, (understanding simple interest), Q8 (understanding compound interest), and Q9 & Q10 (understanding the difference between simple and compound interest). Figure 7 shows a general overall improvement after the interventions compared to the data obtained at the beginning **(Fig. 3)**

Figure 6

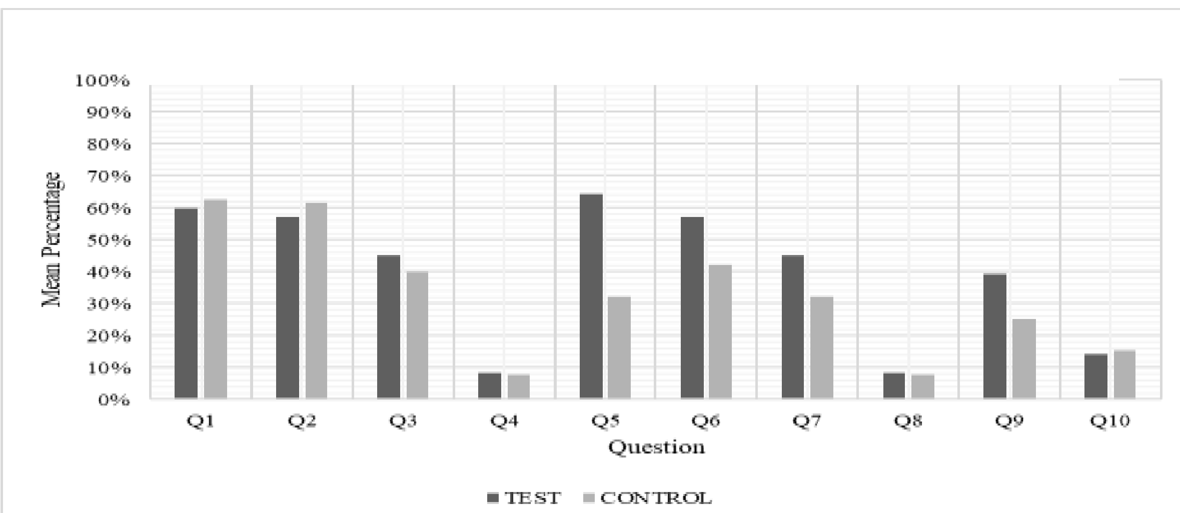
Answers to a Modelling Task from Students in the Test Group



Note: *Excerpts from modelling task from students in the test group show a better understanding of the compound interest model than what the students presented in the pre-test.*

Figure 7

Graph Showing Performance in Post-Test Items in the Intervention and Control Group



Note: Graph comparing the performance of intervention group and the control group in each question in the post-test items. In Fig. 3, the Intervention group is labelled TEST, and the control group is labelled CONTROL.

Question 4 (Q4) and Q8, both coded as “mechanical/procedural fluency” both required students to change the subject of a formula. There was also an improvement in understanding simple interest (Category 1 questions) with minor difference between the two groups. We performed a Mann-Whitney Test to check if the mean scores in the two groups were significantly different. Preliminary finding revealed no significant difference between mean scores of the test group (M=42,7%) and the mean score of control group (M=32,3%) (See Table 3).

Table 3

Mann-Whitney Test for the Difference Between Two Means.

Variable	Observations	Obs. with missing	Obs. without	Minimum	Maximum	Mean	Std. deviation
Test	28	0	28	0,037	0,815	0,427	0,245
Control	26	0	26	0,037	0,815	0,323	0,201

Mann-Whitney test / Two-tailed test:

U	454,500
U (standardized)	1,561
Expected value	364
Variance (U)	3326,110
p-value (Two-tailed)	0,119
alpha	0,05

The exact p-value could not be computed. An approximation has been used to compute the p-value.

Test interpretation:
H0: The difference of location between the samples is equal to 0.
Ha: The difference of location between the samples is different from 0.

As the computed p-value is greater than the significance level alpha=0,05, one cannot reject the null hypothesis H0.

This finding, however, did not account for the unequal observed performance in the Category 2 questions on compound interest. Therefore, Category 2 questions were excluded from the rest of the data and the mean scores from both groups were subjected to an independent Mann-Whitney Test.

The Mann-Whitney Test revealed that the intervention group (Mean=46%, SD=2.81) was significantly different from the control group (Mean=27%, SD=2.65), t (52) =2.58, p<.013, $d = .70$. Using Cohen's (1988) estimates of the effect size d, values of $d =$ .2, .5, and .8 correspond to small, medium, and large effect sizes, respectively. The effect size of .70 is medium to large size in practice, indicating that the grade 10 learners who were taught based on modelling and guided discovery learning principles reported a mean score that was .70 standard deviations higher than the group that was taught based on direct instruction. This finding showed that although the overall performance in the post-test items appears to be the same, there was in fact a significant improvement in the understanding of compound interest in the intervention group than in the control group.

The specific guidance received by students in the control and test groups varied by design, according to whether a student belonged to the guided discovery and modelling group (test group), or to the explicit instruction (control group). Apart from the classical teaching approach used in the control group with occasional explaining of meaning of terms and showing students "how to" solve problems, students in the control group did not receive any of the learning assistance that those in the test group received, such as the teacher explaining the meaning of terms to the students, providing hints to problem solving but having students to solve the problems themselves, as well as providing hints to the students to change course if the teacher sensed that they were on a wrong track (Fig. 9).

Table 4

Mann-Whitney Test for the Difference Between Two Means.

Summary statistics:

Variable	Observations	Obs. with missing	Obs. without	Minimum	Maximum	Mean	Std. deviation
Test	28	0	28	0,000	0,923	0,464	0,281
Control G	26	0	26	0,000	1,000	0,218	0,364

Mann-Whitney test / Two-tailed test:

U	557
U (standa	3,426
Expected	364
Variance (	3156,320
p-value (T	**0,001**
alpha	0,05

The exact p-value could not be computed. An approximation has been used to compute the p-value.

Test interpretation:

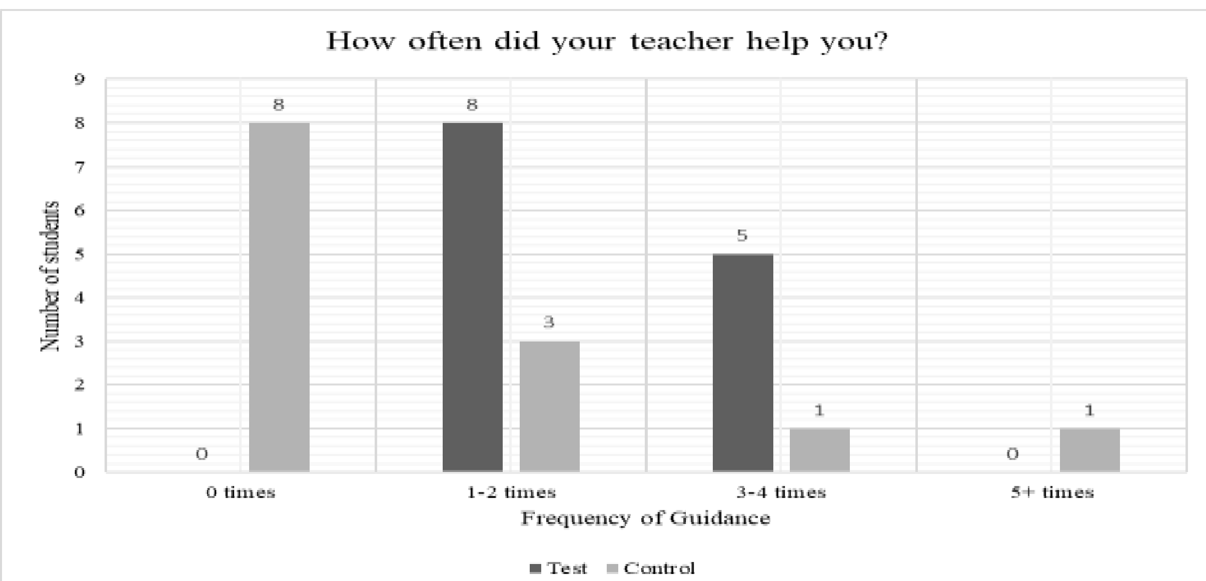
H0: The difference of location between the samples is equal to 0.

Ha: The difference of location between the samples is different from 0.

As the computed p-value is lower than the significance level alpha=0,05, one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.

Figure 8

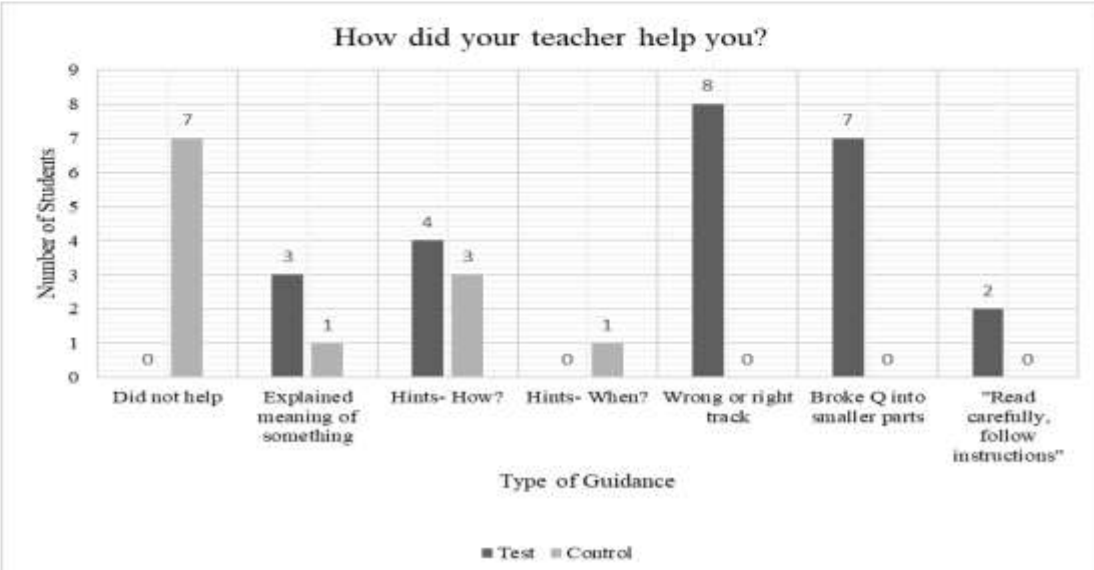
Graph Showing the How Often the Teachers in the Control and Test Groups Assisted Students with the Modelling Task:



Note: The frequency of guidance that students received during modelling task. None of the students in the test group reported having completely no (zero) assistance from their teacher at any single time.

Figure 9

Graph Showing the Specific Guidance Students from the Control and Test Groups Received



Discussions

The modelling tasks were designed on the assumption that students had gained adequate knowledge from their pervious grades 8 and 9 as well as the first half of grade 10. However, from the findings, it was evident that students did not have a good understanding of the linear function and factorising by taking out the highest common factor. These algebraic gaps hindered the learning and discovery meant to take place in the modelling task. Such algebraic gaps evident both in pre-and to a less extent post-test results, included substitution into a formula and manipulating an algebraic equation. Students in both the intervention and control groups performed similarly in those questions obtaining low mean scores. Zulkarnean (2018) has shown that students' conceptual understanding and procedural skills in areas of mathematics must be considered before setting a modelling task. Given that the researchers in this study were from outside the school and therefore lacked adequate background of the school teaching arrangements, they designed the modelling tasks based on an assumption that students were prepared with adequate conceptual understanding and procedural skills that students at that grade *should* have. However, we later discovered during the learning sessions that students faced difficulties deriving the simple interest formula from the linear graphs that they had plotted, a gap which revealed their difficulties with the concept of simple interest.

Zulkarnean's (2018) work about students not understanding a problem, particularly a modelling problem, states that students "are likely to make a guess without having any mathematical thinking process" (p. 5). Although the mean scores between both groups in the post-test differed by 11%, a two-sample independent t-test showed that the difference was not statistically significant. This would mean that there is no difference in the knowledge gained using direct instruction methods and doing a modelling task using the guided discovery approach. However, a closer look at the data gave us an interesting finding. By dividing the post test questions into categories, it came to our attention that the test group had in fact performed much better in compound interest (Category 2 questions) than the control group. Compound interest was new and challenging concept for both groups. Understanding the difference between simple and compound interest was also a major modelling challenge for the students in both groups at the beginning. However, in the post-test data, the mean scores for Category 2 questions differed significantly with the intervention group scoring 19 percentage points above the control group. A t-test further showed that there was in fact a significant difference in mean scores for questions relating to the understanding of compound interest. It was clear, when reviewing the modelling tasks after the pre-test, that students in both the intervention and control groups had similar misconceptions. However, from what our data revealed after the interventions, it became clear that misconceptions about simple and compound interest were not rectified in the control group. This gap is evident in the post-test results which showed a significant difference in the students' understanding of compound interest. The t-test also provided supporting evidence that the intervention group obtained a significantly higher score in their understanding compound interest than the control group. Our study confirms the literature, in that the conceptual understanding is improved with guidance and even more accurately, guidance in the line of *scaffolding*, *heuristics* and *direct presentation of information* (De Jong & Lazonder, 2014).

We briefly read through the teacher questionnaire to find out what worked and what did not work with the intervention teacher (Ms. Cameron) regarding the three intervention principles (heuristics, scaffolds, and direct presentation) that she adopted in her teaching. In the teacher questionnaire, Ms. Cameron stated that *scaffolds* were the most difficult of the three principles to implement, *"[It] takes more time but it [is] a good discovery principle. It can be time consuming as they (the students) have forgotten. As the teacher breaks down the question, [it] must be re-explained."* Ms. Cameron's main difficulty with scaffolds was the time it took for students to retain the skills that they had previously learnt, which they needed in the discovery exercise. As Ms. Cameron was scaffolding, and breaking up the question into smaller pieces, she found herself having to reteach skills, such as how to plot coordinate points to allow students to progress to the next step in their discovery.

Ms.... Cameron also frequently intervened, with her popular type of intervention, which was "breaking the question down into smaller parts." We take this as evidence of her frequent use of scaffolding her lessons. She also often "explained the meaning of something" which is evidence of *direct presentation of information* as well as some form of *heuristics*. However, no learners in the test group said that Ms. Cameron gave hints on when to do something to solve a given task. It is more evident that Ms. Cameron used prompts, rather than heuristics. Ms. Cameron was very aware of her role as a guide or facilitator as she frequently reassured or warned students about whether they were on the right track. From her perspective, when asked which guided discovery principal she used most often, she said she used heuristics most often followed by scaffolds and then direct presentation of information.

Conclusion and Limitations

The research highlighted the contributions of mathematical modelling and guided discovery learning approaches over explicit instruction, especially in promoting the understanding of difficult concepts in financial mathematics, and mathematics in general. We recommend that attention is given to the teaching of basic skills (such as percentage increase, plotting coordinate points, substituting into, and using a formula) at the lower high school grades as a pre-requisite for better up-take of challenging concepts at the upper grade levels. We are convinced that if foundation skills are adequately covered at the lower grades, more time will be freed at the higher grades for students to learn new concepts and higher-level skills instead of spending such valuable time on making up for basic skills which they should already have mastered before. The sample size of sample of 54 students in this study limit the extent to which we can generalise the findings, but the statistical procedure used was appropriate and matched the data. Limitations aside, the current study contributes to an important and on-going discussions on improving the quality of instruction in the classroom, to maximise learning and applying of concepts in real life situations by students. Mathematical modelling and the associated constructivist approaches to teaching and learning are central to such on-discussions. We recommend that the study be replicated in a larger study involving more schools, and that the teaching interventions are extended over a longer period of at least three months.

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